



THE EFFECT OF THE OVERALL HEAT TRANSFER COEFFICIENT VARIATION ON THE OPTIMAL DISTRIBUTION OF THE HEAT TRANSFER SURFACE CONDUCTANCE OR AREA IN A STIRLING ENGINE

M. COSTEA^{†*} and M. FEIDT[‡]

[†]Department of Applied Thermodynamics, Polytechnic University of Bucharest, Splaiul Independentei 313, Bucharest, Romania

[‡]L.E.M.T.A., University H. Poincaré of Nancy 1, C.N.R.S. URA 875, 2, avenue de la Forêt de Haye, 54516 Vandoeuvre-les-Nancy, France

Abstract—This study aims to assess for a Stirling engine the influence of the overall heat transfer coefficient variation on the optimum state and on the optimum distribution of the heat transfer surface conductance or area among the machine heat exchangers. The analysis is based on a Stirling machine optimization method, previously elaborated, which is now applied to a cycle with total heat regeneration. The method was conceived for an irreversible cycle with heat transfer across temperature differences at the source and the sink, and heat losses between the hot-end and the cold-end of the engine. Source and sink of finite thermal capacity as well as thermostats are considered. The new approach considers a linear variation of the overall heat transfer coefficient of the machine heat exchangers with respect to the local temperature difference. A comparison of the optimum state and the optimum distribution of the heat transfer surface conductance or area among the heater and the cooler is made for several cases. © 1998 Elsevier Science Ltd. All rights reserved

Stirling engine Optimization method Irreversible cycle Heat transfer surface Conductance
or area distribution

NOMENCLATURE

A = Heat transfer surface area
 $\dot{C}$ = Capacity flowrate
 $\bar{h}$ = Overall heat transfer coefficient
 $\bar{h}A$ = Heat transfer surface conductance
 k' = Proportional factor
 NTU = Number of heat transfer units
 $\dot{Q}$ = Heat transfer rate
 T = Temperature
 $\dot{W}$ = Power delivery

Greek characters

ϕ = Heat transfer rate at the source
 γ = Ratio of specific heats (c_p/c_v)
 η = Cycle efficiency
 θ = Temperature ratio

Subscripts

c = Cold
 f = Working fluid of the cycle
 H = Hot end, source
 i = Intermediary
 L = Cold end, sink
 loss = Related to heat losses
 min = Minimum
 r = Reduced
 S = Generally for the source or sink (as in T_S)
 T = Related to the total heat transfer surface conductance or area
 w = Warm
 0 = Reference
 1 = In
 2 = Out

*To whom all correspondence should be addressed. Fax: 40-1-410.44.88; E-mail: mcos@theta.termo.pub.ro.

INTRODUCTION

In the last few years, the Stirling cycle machine has been intensively studied in connection to its possible solar, extra-terrestrial and military applications. Recent papers have made major contributions to the study and optimization of the Stirling machine [1–4].

Following the current trend, the authors have elaborated a Stirling engine optimization method [5], according to which the cycle was optimized with respect to the maximum power delivery. The method was conceived to provide the engine optimum state defined by the outlet temperatures of the sources, the temperatures of the working fluid during the isothermal heat input and output of the cycle, and the cycle efficiency. The optimum distribution of the heat transfer surface conductance or area among the engine heat exchangers were also obtained.

The originality of this model came from the fact that the heat transfer rate available at the source is treated as a parameter, as opposed to past studies where the parameter was the source temperature.

The authors extend here previous works on Stirling engines [6, 7] by considering the influence of the overall heat transfer coefficient variation on the engine characteristics, particularly on the distribution of the heat transfer surface conductance or area among the heater and the cooler.

MODELING AND OPTIMIZATION OF THE STIRLING ENGINE IRREVERSIBLE CYCLE

The isothermal Stirling engine cycle with total heat regeneration is presented in Fig. 1. The continuous variation of the temperature difference between the source/sink and the cycle working fluid suggests the finite thermal capacity of the source and the sink. The thermal reservoirs become thermostats when this $\Delta T \rightarrow \infty$. Also this case is analyzed in the paper. The external irreversibilities of the cycle are represented in this study by:

- the heat input rate $\dot{Q}_H$ across $\Delta T_H = \Delta T_{Hi} - T_w$ at the source and the heat output rate $\dot{Q}_L$ across $\Delta T_L = T_{Li} - T_c$ at the sink;
- the heat loss, $\dot{Q}_{loss}$, between the hot-end (T_{H1}) and the cold-end (T_{L1}) of the engine.

The model developed by Costea [6] is extended here to show the influence of the overall heat transfer coefficient variation on the performance of the engine. Generally, the overall heat transfer coefficient of the heat exchanger is not constant with respect to the temperature variation. Actually, the overall heat transfer coefficient may be considered as any function of ΔT due to the isothermal heat transfer input/output in a Stirling machine.

The general case of the non-linear dependency of the overall heat transfer coefficient, $\bar{h}$, on the temperature variation is difficult to model, despite the fact that any variation can be brought

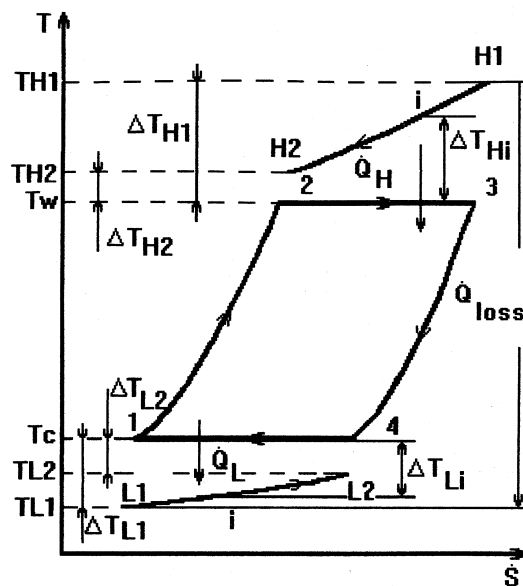


Fig. 1. Temperature-entropy diagram of a Stirling engine cycle with source and sink of finite thermal capacity.

Table 1. Presentation of the studied cases in the present optimization of the Stirling engine

Source and sink of															
FTC—finite thermal capacity								CT—constant temperature							
Flux—fixed heat transfer rate at the source				Temp—fixed source inlet temp.				Flux—fixed heat transfer rate at the source				Temp—fixed source inlet temperature			
WL— with heat losses				NL— without heat losses				WL—with heat losses				NL—without heat losses			
ext†	ext‡	ext	ext	int‡	int	int	int	ext	ext	ext	ext	int	int	int	int
α	β	δ	α	β	δ	α	β	δ	α	β	δ	α	β	δ	α

ext† corresponds to $\dot{C}_{\text{ext}} = \min(\dot{C}_{\text{ext}}, \dot{C})$, where $\dot{C}_{\text{ext}} = (\dot{C}_H, \dot{C}_L)$, with the assumption $\dot{C}_H = \min(\dot{C}_H, \dot{C}_L)$.

int‡ corresponds to $\dot{C} = \min(\dot{C}_{\text{ext}}, \dot{C})$, and α, β, δ are given by the following constraints:

(α) Fixed heat transfer surface conductance of the heater and the cooler

$$\bar{h}_{L0}A_H, \bar{h}_{L0}A_L = \text{fixed}.$$

(β) Fixed total heat transfer surface conductance

$$\bar{h}_{H0}A_H + \bar{h}_{L0}A_L = \text{fixed} = \bar{h}_{T0}A_T.$$

(δ) Fixed total heat transfer surface area

to a linear one within small intervals. Colburn, for example, assumed a linear variation of $\bar{h}$ with respect to the local temperature difference in the heat exchanger [8], as follows:

$$\bar{h}_{Si} = \bar{h}_{S0} \cdot (1 + \bar{h}'_{Si} \cdot \Delta T_{Si}) \quad (1)$$

where $\bar{h}_{S0}$ is a reference value for the overall heat transfer coefficient of the source/sink heat exchanger, and $\bar{h}'_{Si}$ is the proportional factor of this variation.

The authors of the present study adopted the above assumption as a new approach in Stirling engine theoretical investigation.

The heat losses between the source and the sink are included in the analysis via an additional heat transfer conductance, and is expressed by equation (2):

$$\dot{Q}_{\text{loss}} = (\bar{h}A)_{\text{loss}} \cdot (T_{H1} - T_{L1}). \quad (2)$$

The mathematical model is based on the First and Second Law of Thermodynamics applied to the cycle, the energy balance at the source, and heat transfer equations. All the aforementioned irreversibilities are included. The model consists of a nonlinear system of equations (objective function and two constrains). Solutions to the model are found by using Lagrange's multiplier method.

For each set of assumptions (see Table 1), the model yields different expressions for the objective functions and the constraints. Two representative cases were chosen to illustrate the resulting system of equations:

1. Source and the sink of finite thermal capacity: FTC—flux—WL—ext— α (see Table 1).

(i) The objective function:

$$\dot{W}_r = -\frac{\gamma \dot{C}_H}{\dot{C}} \Delta \theta_{H1} \left[1 - \frac{\exp(-NTU_{H0})}{1 + k'_{Hr} \Delta \theta_{H1} [1 - \exp(-NTU_{H0})]} \right] \cdot \left(1 - \frac{1 - \Delta \theta_{L1}}{\theta_{H1} - \Delta \theta_{H1}} \right). \quad (3)$$

(ii) The constraints:

• the cycle entropy generation

$$\frac{\frac{\gamma \dot{C}_H}{\dot{C}} \Delta \theta_{H1} \left[1 - \frac{\exp(-NTU_{H0})}{1 + k'_{Hr} \Delta \theta_{H1} [1 - \exp(-NTU_{H0})]} \right]}{\theta_{H1} - \Delta \theta_{H1}} + \frac{\frac{\gamma \dot{C}_L}{\dot{C}} \Delta \theta_{L1} \left[1 - \frac{\exp(-NTU_{L0})}{1 + k'_{Lr} \Delta \theta_{L1} [1 - \exp(-NTU_{L0})]} \right]}{1 - \Delta \theta_{L1}} = 0 \quad (4)$$

- the energy balance at the source

$$\frac{\gamma \dot{C}_H}{\dot{C}} \Delta \theta_{H1} \left[1 - \frac{\exp(-NTU_{H0})}{1 + k'_{Hr} \Delta \theta_{H1} [1 - \exp(-NTU_{H0})]} \right] - \left[\dot{\phi}_r - \frac{\gamma \dot{C}_H}{\dot{C}} NTU_{\text{loss}} (\theta_{H1} - 1) \right] = 0. \quad (5)$$

(2) Source and the sink of constant temperature: CT-temp-ext- α .

(i) The objective function:

$$\dot{W}_r = -\frac{\gamma \dot{C}_H}{\dot{C}} NTU_{H0} (1 + k'_{Hr} \Delta \theta_H) \Delta \theta_H \left(1 - \frac{1 - \Delta \theta_L}{\theta_H - \Delta \theta_H} \right). \quad (6)$$

(ii) The constraint:

- the cycle entropy generation

$$\frac{\frac{\gamma \dot{C}_H}{\dot{C}} NTU_{H0} \Delta \theta_H (1 + k'_{Hr} \Delta \theta_H)}{\theta_H - \Delta \theta_H} + \frac{\frac{\gamma \dot{C}_L}{\dot{C}} NTU_{L0} \Delta \theta_L (1 + k'_{Lr} \Delta \theta_L)}{1 - \Delta \theta_L} = 0. \quad (7)$$

Note that both systems are representative for the case with fixed heat transfer surface conductance corresponding to the reference value of the overall heat transfer coefficient ($\bar{h}_{H0} A_H$ and $\bar{h}_{L0} A_L$) of the heater and the cooler. For each of the other two cases— β and δ —the NTU_{H0} and the NTU_{L0} will be modified corresponding to the new conditions imposed by the new constraint. Also a new variable (heat transfer surface conductance or area) will be added to those of (α)-case.

The non-dimensional variables of the model are:

—the temperatures:

$$\text{FTC} \quad \Delta \theta_{L1} = 1 - \frac{T_C}{T_{L1}}, \quad \Delta \theta_{H1} = \frac{T_{H1} - T_w}{T_{L1}}, \quad \theta_{H1} = \frac{T_{H1}}{T_{L1}}$$

—the heat transfer surface conductances or areas:

$$(\beta) \quad \frac{\bar{h}_{H0} A_H}{\bar{h}_{T0} A_T}, \quad \frac{\bar{h}_{L0} A_L}{\bar{h}_{T0} A_T} \quad (\delta) \quad \frac{A_H}{A_T}, \quad \frac{A_L}{A_T}.$$

The model parameters are:

—the total reference heat transfer conductance equivalent NTU:

$$\diamond_{\text{ext}} : NTU_{T0} = \frac{\bar{h}_{T0} A_T}{\dot{C}_C} \quad \diamond_{\text{int}} : NTU_{T0} = \frac{\bar{h}_{T0} A_T}{\dot{C}}$$

—the proportional factor of the overall heat transfer coefficient variation:

$$\bar{h}'_{Hr} = \frac{\bar{h}'_H}{\bar{h}_{H0}} \cdot T_{L1} \quad \bar{h}'_{Lr} = \frac{\bar{h}'_L}{\bar{h}_{L0}} \cdot T_{L1}$$

—ratio of the reference overall heat transfer coefficient of the sink and the source:

$$(\delta) \quad r\bar{h} = \frac{\bar{h}_{L0}}{\bar{h}_{H0}}$$

—the heat loss conductance equivalent NTU:

$$\diamond_{\text{ext}} : NTU_{\text{loss}} = \frac{(\bar{h}A)_{\text{loss}}}{\dot{C}_C} \quad \diamond_{\text{int}} : NTU_{\text{loss}} = \frac{(\bar{h}A)_{\text{loss}}}{\dot{C}}$$

—the heat transfer rate available to the source:

$$\text{FTC} \quad \dot{\phi}_r = \frac{\dot{\phi}}{\dot{C}/\gamma \cdot T_{L1}} = f_{ir}.$$

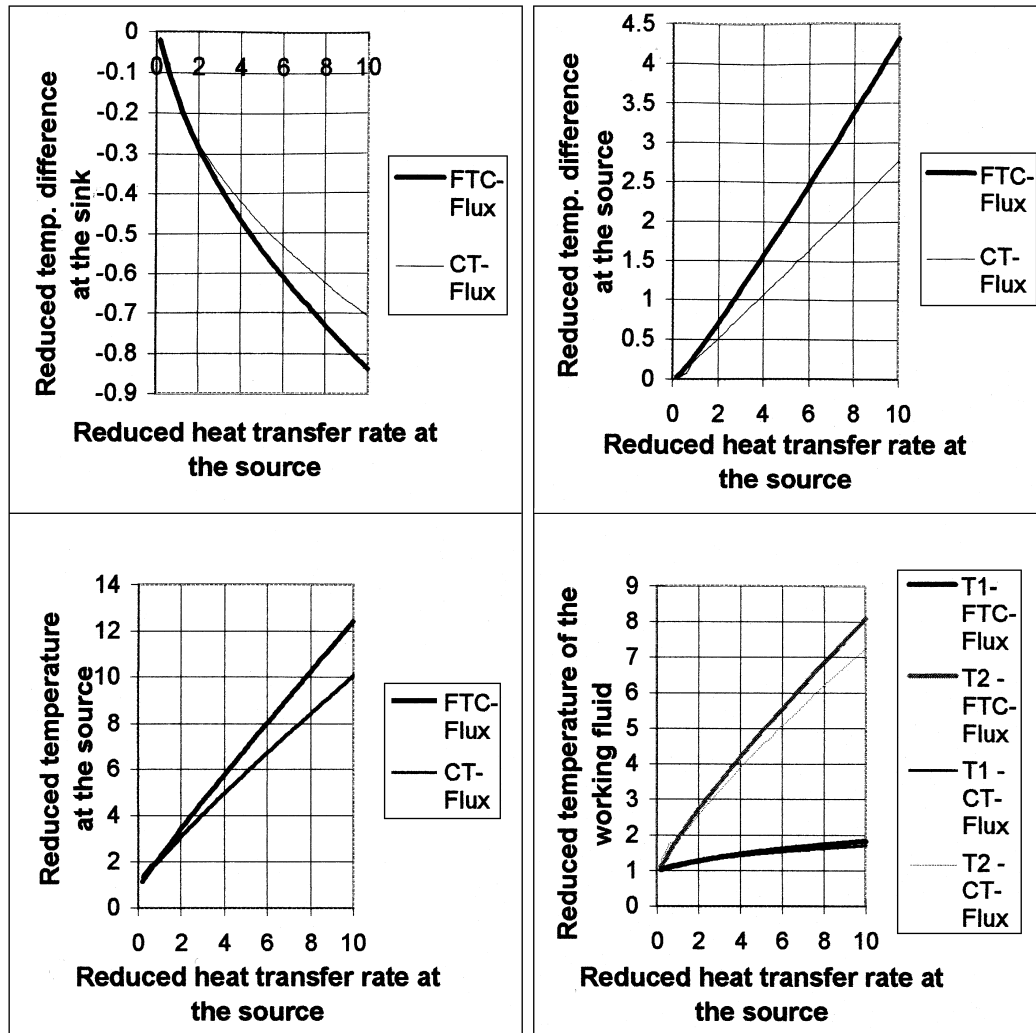


Fig. 2. Influence of the heat transfer rate available at the source on the Stirling engine optimum temperatures for finite thermal capacity and constant temperature of the source

$$(\theta_H = 3; \quad NTU_{T0} = 5; \quad NTU_{loss} = 0.5; \quad \bar{h}_{Hr} = 0.1; \quad \bar{h}_{Lr} = 0.1).$$

Note that T_{L1} from the above FTC case equations becomes T_L in those corresponding to the CT case.

RESULTS AND DISCUSSION

The summary of the possible cases to be studied with this optimization model is presented in Table 1. Analytical results were obtained only for the cases without heat losses between the source and the sink, and they show that there is no physical optimum.

Note that when the source temperature is fixed, the energy balance at the source cannot be written anymore. Since it was the only equation to account on the heat losses between the source and the sink, there is no more meaning to distinguish two cases corresponding to the heat losses. Just to complete the optimization results, one can calculate separately the total heat transfer rate ($\dot{Q}_H + \dot{Q}_{loss}$) supplied from the source.

Also there is no meaning to consider $\dot{C} = \min(\dot{C}, \dot{C}_{ext})$ for the case of finite thermal capacity of the source and the sink because of the isothermal processes of cycle heat input/output.

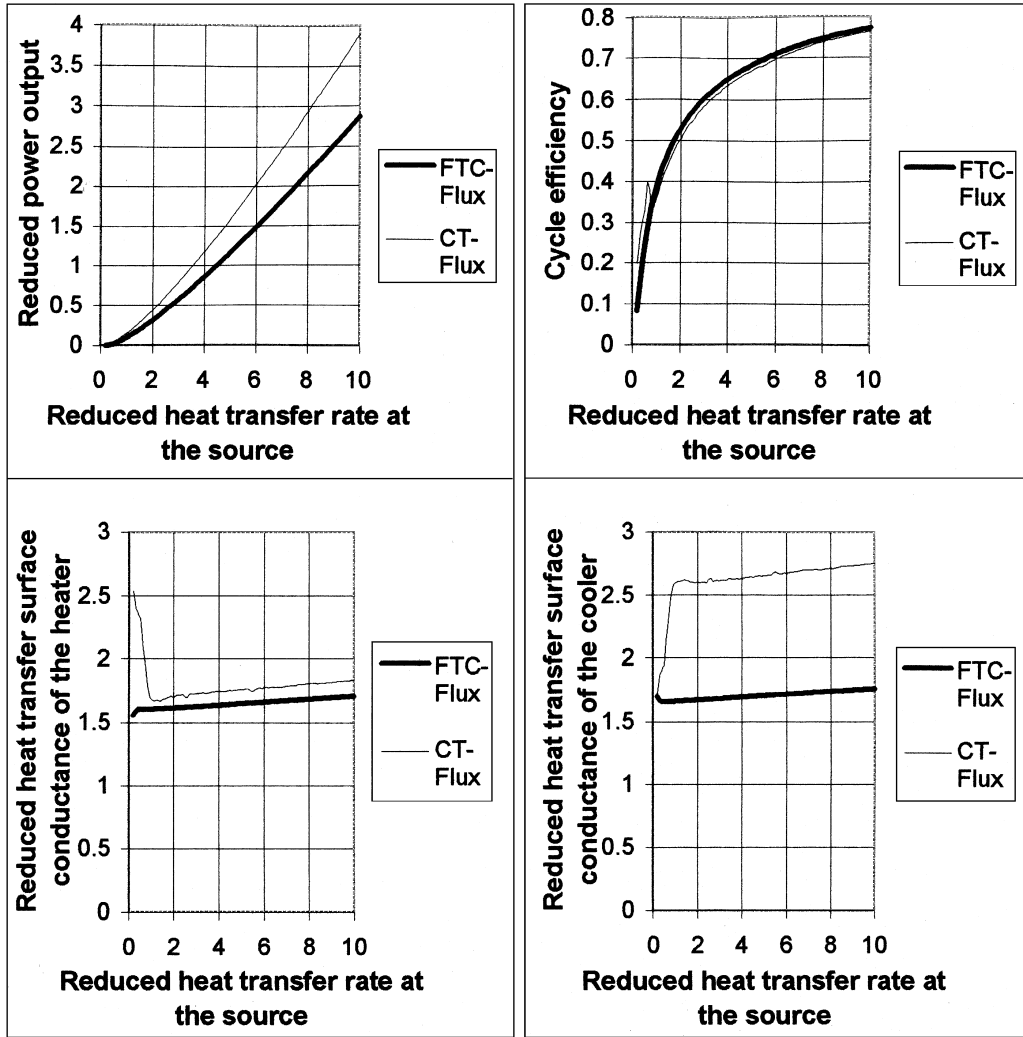


Fig. 3. Influence of the heat transfer rate available at the source on the Stirling engine power output, cycle efficiency, and optimum heat transfer surface conductance for finite thermal capacity and constant temperature of the source

$$(\theta_H = 3; \quad NTU_{T0} = 5; \quad NTU_{loss} = 0.5; \quad \bar{h}_{Hr} = 0.1; \quad \bar{h}_{Lr} = 0.1).$$

A parametric study was conducted in order to observe the effect of the overall heat transfer coefficient variation on the optimum state, and on the optimum distribution of the heat transfer surface conductance or area among the engine heat exchangers.

We attempted to provide realistic central values for the model parameters, as well as for their variation range. In order to compare the results with previous data obtained for Stirling cycle solar application, the central value of the heat transfer rate available to the source was chosen high. The present investigation considered the following parameter ranges:

- $\phi_r = (0.1, 10)$, central value 0.75 or $\theta_{H1} (1.5, 8)$, central value 3;
- $NTU_{T0} = (0.1, 5)$, central value 3;
- $NTU_{loss} = (0, 5)$, central value 0.5;
- $\bar{h}'_{Hr}, \bar{h}'_{Lr} = (0, 1)$, central value 0.1;
- $r\bar{h} = (0.01, 4)$, central value 0.1.

Some selected results are graphically presented in Figs 2–5.

Figures 2 and 3 show that, for the case of source and sink of constant temperature, the temperature differences, $\Delta\theta^*_{H1}$, $\Delta\theta^*_{L1}$, decrease and thus the power output will increase relative to

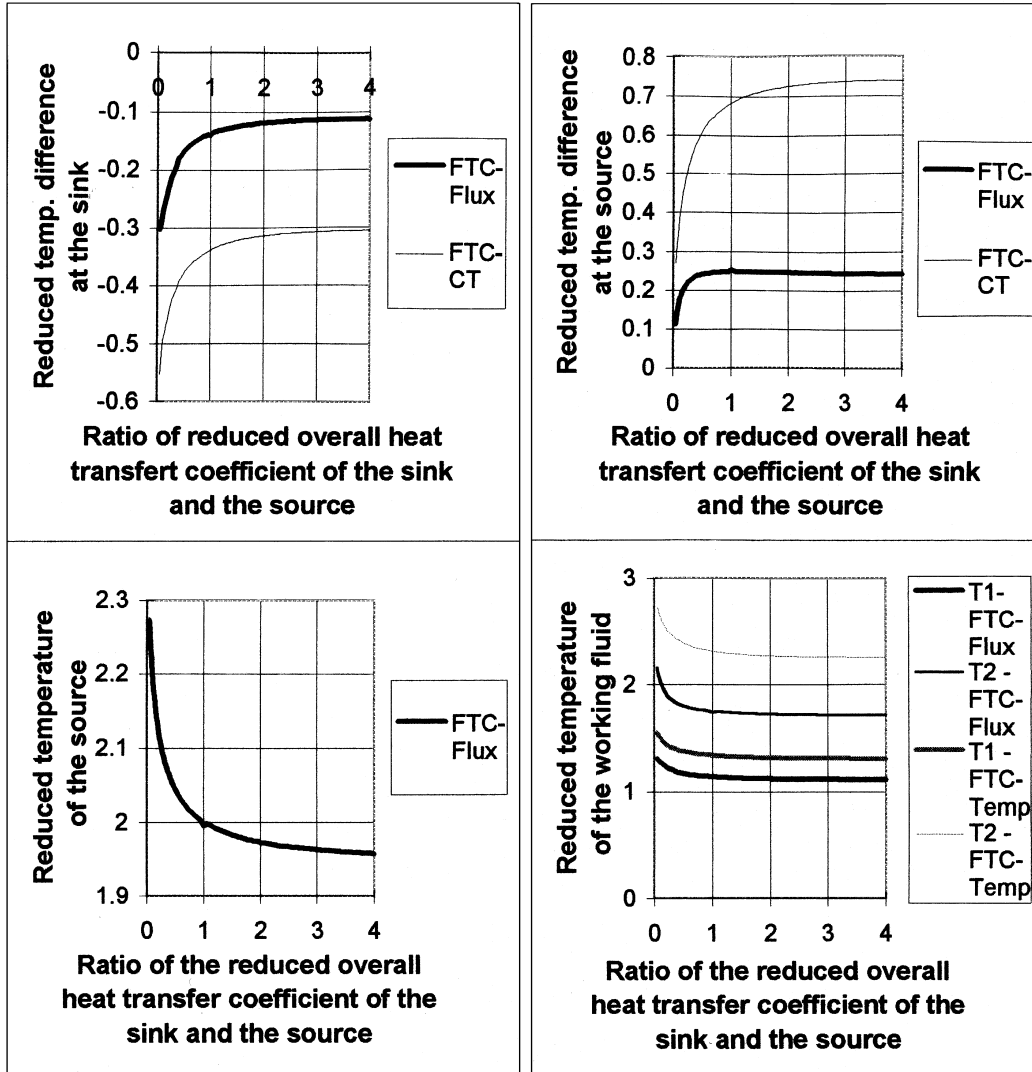


Fig. 4. Comparison of the engine temperatures for the case of finite thermal capacity of the source and sink, with heat transfer rate available at the source or fixed inlet source temperature

$$(\text{fir} = 0.75 \text{ or } \theta_{H1} = 3; \quad \text{NTU}_{T0} = 5; \quad \text{NTU}_{\text{loss}} = 0.5; \quad \bar{h}_{Hr} = 0.1; \quad r\bar{h} = 0.1).$$

the corresponding values of the FTC case. The cycle efficiency and the working fluid cold-end temperature are not affected. Significant differences between the results of these cases are registered for high value of ϕ_r . However, for small values of ϕ_r , only the heat transfer conductances of the heater and cooler are significantly sensitive with respect to the heat transfer rate variation (decreasing from 2.5 to 1.7 for the heater, increasing from 1.5 to 2.65 for the cooler, when ϕ_r increase from 0.1 to 1).

Results in Figs 4 and 5 show that an imposed heat transfer rate at the source yields smaller value of the source inlet temperature comparatively to $\theta_H = 3$ of the FTC-temp case. As a consequence, the power output of the engine decreases drastically by about 300%. The associated heat transfer surface area distribution was expected, since an increase in $r\bar{h}$ means $\bar{h}_{L0} \uparrow$ and $\bar{h}_{H0} \downarrow$, namely, the opposite variation of the corresponding areas.

Remarkably, there is no difference between the heat transfer surface area distribution for these two cases. As a final remark on Fig. 3, it is obvious that is worthless to enhance very much the overall heat transfer coefficient of the cooler, as the important variation of the cycle characteristics are registered for $r\bar{h} = (0.01, 2)$.

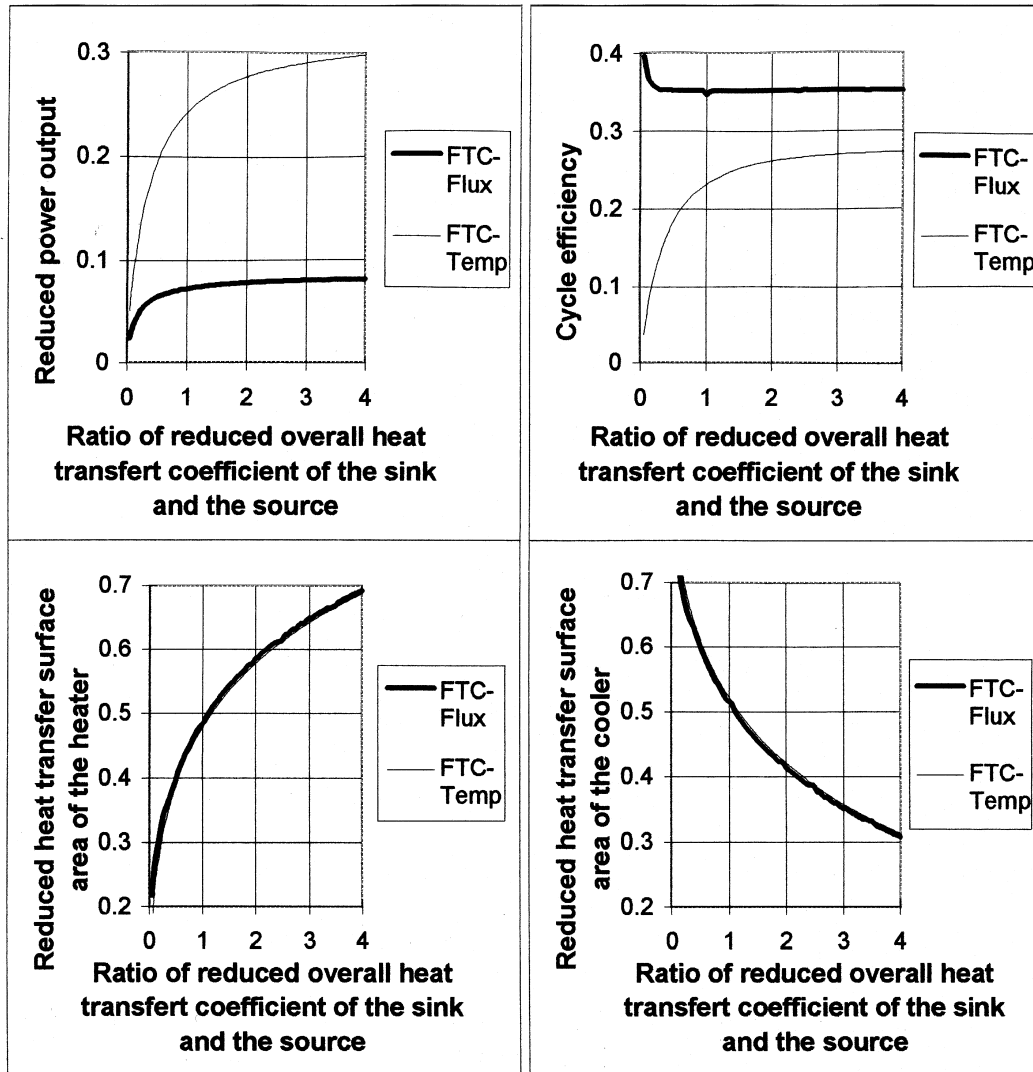


Fig. 5. Comparison of the engine power output, cycle efficiency, and optimum heat transfer surface area distribution for the case of finite thermal capacity of the source and sink, with heat transfer rate available at the source or fixed inlet source temperature

$$(\text{fir} = 0.75 \text{ or } \theta_{HI} = 3; \quad \text{NTU}_{T0} = 5; \quad \text{NTU}_{\text{loss}} = 0.5; \quad \bar{h}_{Hr} = 0.1; \quad r\bar{h} = 0.1).$$

CONCLUSIONS

A theoretical model was presented for the Stirling engine cycle optimization based on a linear variation of the overall heat transfer coefficient with the temperature difference. Solutions corresponding to different sets of assumptions and parameter values were used for a comparison of the engine characteristics. Results pointed out either optimal variation range for some model parameters, or some significant differences of the power output, source and sink temperature difference, heat transfer characteristics values with respect to each of the studied cases. The model allows further investigation that can provide additional insight into the optimal design of a Stirling engine, being currently developed by the authors.

REFERENCES

1. Walker, G., in *Cryocoolers—Part 1: Fundamentals, Part 2: Applications*. Plenum Press, New York, 1983.
2. Organ, A. J., in *Thermodynamics and Gas Dynamics of Stirling Cycle Machine*. Cambridge University Press, 1992.
3. Atrey, M. D., Bapat, S. L. and Narayankhedkar, K. G., *Cryogenics*, 1954, **33**(10), 951.

4. Walker, G., Reader, G., Fauvel, O. R. and Bingham, E. R., in *The Stirling Alternative—Power Systems, Refrigerants and Heat Pumps*. Gordon and Breach Science Publishers, Switzerland, 1994.
5. Costea, M., in *Internal Report, LEMTA*. University H. Poincaré of Nancy 1, Nancy, France, 1994.
6. Costea, M., Feidt, M. and Belabbas, B., in *Proc. of the Inter. Sympos. on Energy Systems and Design Analysis, ESDA '96*, Montpellier, France, 1996, pp. 207–214.
7. Feidt, M., Costea, M. and Belabbas, B., in *Proc. of the Inter. Sympos. on Efficiency, Costs, Optimization Simulation and Environmental Aspects of Energy Systems, ECOS '96*, Stockholm, Sweden, 1996, pp. 99–103.
8. Bouvenot, A., in *Transferts de chaleur*. Masson, Paris, 1981.